Pre-heating in the framework of massive gravity

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Abstract

In this paper we propose a mechanism of natural preheating of our universe by introducing an inflaton field dependent mass term for the gravitational wave for a specific class of massive gravity theory. For any single field inflationary model, the inflaton must go through the oscillatory phase after the end of inflation. As has recently been pointed out, if the gravitational fluctuation has inflaton dependent mass term, there will be a resonant amplification of the amplitude of the gravitational wave during the oscillatory phase of inflaton. Because of this large enhancement of the amplitude of the gravitational wave due to parametric resonance, we show that universe can naturally go through the pre-reheated phase with minimally coupled matter field. Therefore, during the reheating phase, there is no need to introduce any arbitrary coupling between the matter field and the inflaton .

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I. INTRODUCTION

During the last decade, observational cosmology [1] has reached to a level, through which we can go backward in time, and see our universe with a great precession almost near the beginning of our universe. By now, we have understood very well the mechanism of how the observed very tiny fluctuation ($\delta T/T \simeq 10^{-5}$) in temperature in the Cosmic Micro-wave Background (CMB) sky plays the roll of seed field for the structure of the current universe. So far inflation, which is believed to have happened within a very short period of time after the big-bang, is believed to be the best theoretical explanation of the origin of those tiny fluctuation. The inflation was first proposed [2–4] to explain some important conceptual problems persisted in the standard hot big-bang cosmology. In spite of its simplicity, it was soon realised that constructing such a model within the known theory is very difficult. Main problems in constructing an inflation model are the mechanism of how it has happened, the end of inflation, how the universe gets reheated after such a super cooling phenomena, and most importantly the origin of inflaton itself. In addition to all those, there generally exist many issues within a particular model under consideration. Living with many fundamental questions, in this paper we will ask the question as to how reheating happened after the end of inflation. Immediately after the proposal of inflation, this question had naturally emerged and studied in detail. There exist lot of studies on this particular mechanism [5, 6], which we generally call pre-heating. In this paper we will introduce a new mechanism in the framework of a certain class of gravity theories where the gravitational fluctuation acquire inflaton field dependent effective mass term either from the original theory such as massive gravity theory or from some higher derivative gravity theory. In this paper we will be focussing on a class of massive gravity theory proposed in [7]. In the aforementioned class of theories our mechanism will be generic in the sense that for any inflationary model, after the end of inflation, the inflaton field generically oscillates, which in turn will provide the oscillating mass term for the graviton, and resonant amplification in the matter sector will naturally happen without invoking any further coupling parameters. In order to realise this, in this paper we will consider a class of ghost free massive gravity model [7], where we only have one extra degree of freedom in addition to the usual two tensor polarization modes. And we will identify the additional degree of freedom to be the inflaton. We will see that in this class of models the gravitational fluctuation gets mass which is dependent on that extra scalar degree of freedom. The general physical mechanism to transfer the energy

during pre-heating period is to couple the matter field as a probe with the background oscillating inflaton field. In the usual preheating scenario, one generally couples the matter fields with the inflaton field in an *ad hoc* manner, and try to tune those unknown coupling constant to have explosive particle production through parametric resonance. Here we will show that because of inflaton dependent mass term for the graviton fluctuation how the universal minimal coupling of the matter field with the gravity is sufficient to reheat our universal after the end of inflation. The idea was first put forward in the reference [8], where the inflaton field is assumed to be coupled with the higher derivative gravitational Chern-Simons term. However, as we have explained in [17], it is very difficult to have successful preheating with the Chern-Simons coupling. It was shown to be crucially dependent upon the large initial amplitude of the gravitational fluctuation during reheating. However, this large value of initial amplitude is found to be very difficult to achieve. Thanks to the reference [9], which pointed out the possibility of generating large amplitude of the gravitational wave by parametric resonance considering the inflaton field dependent mass term of the graviton. Purpose of the proposal was to create large scalar to tensor ratio without violating Lyth bound [10]. We are interested in preheating. Therefore, to achieve that we will have sequence of parametric resonance phenomena. We will see how the parametric resonance of the gravitational wave naturally leads to the parametric resonance in the matter sector though gravitational coupling. In this paper we will try to consider a minimal model [7] to realise this mechanism.

We structured our paper as follows: in the next section-II we will introduce a class of massive gravity theory, which we will be studying. In the subsequent section-III, we will do the linear perturbation analysis in the gravity and the matter sector. As there in no direct coupling of the matter field with the inflaton, we need to take into account the higher order term in matter field equation. In section-IV, we numerically solve those equations of motion and show that how the non-linear interaction term between the gravitational fluctuation and the matter field is responsible for the explosive particle production. The main driving force behind our mechanism is the successive resonance phenomena. The first resonance happens in the gravitational sector because of the oscillatory mass term and the resonance in the matter field happens in the resonant gravitational background. Because of this resonance in a resonant background, the particle production turned out to be much more efficient in our model than in the usual re-heating scenario. In the final section, we will make some final remarks on our mechanism and also discuss about some future works.

II. MASSIVE GRAVITY MODEL

Massive gravity theory has a long history starting from Fierz-Pauli [11], who first proposed a linearised theory of massive spin two field. Subsequently it has been shown that the linear theory has some pathological behaviours which can not be cured in the linear regime. Therefore, non-linearity has been introduced at the action level in order to have a consistent interacting spin two field theory [12]. One of the interesting facts of the non-linear modification proposed in [12] is its near unique structure when the Lorentz invariance is imposed. But subsequently, the theory has been proved to be difficult to make phenomenologically viable because of its low energy cut off [13]. However, modificantion can be made by suitably modifying the Vainshtein scale to extend its regime of validity [14]. Constructing model would be easer if one relaxes the Lorenz invariance. Therefore, further modification has been proposed to violate Lorentz invariance in a controlled manner such that the theory remains well defined and also becomes cosmologically viable [7, 15, 16]. As we have already mentioned in the introduction, in this paper we will be studying a particular class of models proposed in [7]. We want to demonstrate our mechanism considering the minimal model where we have gravity with one inflaton field. In order to achieve this one considers the following internal symmetries in the goldstone boson sector of gravity

$$\phi'^i = R^i_j \phi^j \quad ; \quad \phi'^i = \phi^i + F^i(\phi^0), \tag{1}$$

where, R_{ij} is the rotation matrix in the field space, and $F^i(\phi^0)$ is arbitrary function of ϕ^0. With this assumption we can write most general action for massive gravity as follows

$$\mathcal{L} = M_p^2 R + \mathcal{L}(X, Y^{ij}, \Box\phi^0, \cdots), \tag{2}$$

where, $\mathcal{L}(X, Y^{ij})$ is the most general action for the goldstone boson fields (ϕ^0, ϕ^i) in accord with the aforementioned internal symmetries. This potential term will contribute to the mass term for the graviton in the unitary gauge. Based on the aforementioned residual symmetries in the goldstone sector, one can write the following scalar quantities which we introduced above in $\mathcal{L}(X, Y^{ij})$,

$$X = g^{\mu\nu}\partial_\mu\phi^0\partial_\nu\phi^0 \quad ; \quad Y^{ij} = g^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j - \frac{(\partial_\mu\phi^0\partial^\mu\phi^i)(\partial_\alpha\phi^0\partial^\alpha\phi^j)}{X}. \tag{3}$$

Since the theory has the symmetry $\phi'^i = \phi^i + F^i(\phi^0)$, by using that one can remove three propagating degrees of freedom from the ϕ^i with its fixed vacuum expectation value $\langle\phi^i\rangle = x^i$.

By using the Hamiltonian analysis also it has been shown that the above theory has only three degrees of freedom. We identify the extra degree of freedom $\phi^0 = \phi$ as an inflaton in our subsequent analysis. For the sake of our numerical computation we will consider the model of inflation from our previous paper [17]. In compatible with the above symmetries we consider the following Lagrangian,

$$\mathcal{L}_{lv}(X, Y^{ij}, \Box\phi, \cdots) = \frac{1}{2}X - \frac{1}{2s^3}M(\phi)X\Box\phi - V(\phi) - \frac{8}{9}M_p^2 m(\phi)^2 \frac{\bar{Y}^{ij}\bar{Y}_{ij}}{Y^2}, \qquad (4)$$

where, we define

$$V(\phi) = \Lambda^4\left(1 - \cos\left(\frac{\phi}{f}\right)\right) \quad ; \quad \bar{Y}^{ij} = Y^{ij} - 3\frac{Y^{ik}Y_k^i}{Y^2} \quad ; \quad Y = \delta_{ij}Y^{ij}. \qquad (5)$$

To the leading order, the crucial difference between the above Lagrangian and the usual massive gravity theory in the literature is the extra scalar field dependent mass term [9]. We are not going to study the theoretical aspects of this kind if mass term in this paper. As one can easily identify, this is the term which plays a crucial role for our mechanism to work. According to our previous discussion, we will study phenomenological aspects of this term in the cosmological context. If one considers the action in unitary gauge, the last term in the above action will play like the mass term of the graviton. s is the dimensionful parameter. $M(\phi)$ is the some specific dimensionless function of the inflaton field. f is the decay constant. Λ can be identified with the scale of inflation. It is also interesting point to note that the mass term will not play the role in the background dynamics.

We would like to emphasize that in this work we are only interested in the regime where the inflaton field coherently oscillates after the end of inflation. To have successful reheating this is one of the important requirements for any model of inflation. Therefore, our final conclusion of this work is valid for any model of inflation. The new ingredient that we have in the Lagrangian is the extra term related to the mass of the graviton. The extra mass term is constructed in such a way that it only contributes to the mass of the gravitational fluctuation. Similar situation happens for gravitational Chern-Simons term coupled with the inflaton. The crucial difference is that the Chern-Simon term provides oscillating anti-dissipation term for the dynamics of the graviton. On the other hand here we have the oscillating mass term. Thanks to the reference [9], in which the authors have already shown that in the oscillating inflaton background the amplitude of the graviton fluctuation undergoes resonant amplification. In this paper will show how that resonant amplification helps to have resonant amplification of the matter field through minimal gravitational coupling. In the subsequent analysis we will show how this mechanism works.

III.   TENSOR PERTURBATION AND PRE-HEATING

In this section we will analyse the tensor and matter perturbation in the oscillating infla-
ton background. As is well known, in the usual pre-heating scenario, in order to create the
matter field from the inflaton, one invokes different types of arbitrary coupling parameters
in the matter Lagrangian,

$$\mathcal{L}_{mat} = \frac{1}{2}\partial_\mu\theta\partial^\mu\theta - m_\theta^2\theta^2 - \frac{1}{2}g^2\phi^2\theta^2 + \frac{1}{2}\chi R\theta^2 + \cdots \tag{6}$$

where, χ and g are the coupling parameters. In the usual Einstein gravity, since there
is no parametric resonance in the gravity sector after inflation, one needs to introduce
above coupling parameters for every matter field to obtain the oscillating mass term for the
matter field under consideration. Parametric resonance will occur because of those coupling
parameters. This in turn will provide us the explosive particle production. What is novel
of our mechanism is that, for the purpose of explosive particle production, we do not need
to introduce any arbitrary coupling constants for every matter fields. We will show in our
following analysis, how minimal gravitation coupling is sufficient for this purpose. The
only parameter that we need in our study is the gravitational mass $m(\phi)$, which we have
introduced before in the massive gravity action. Therefore, for our purpose, we set

$$g = 0 \; ; \; \chi = 0 \tag{7}$$

As we have already mentioned, at the linear order in perturbation, the extra mass term of
the gravitational action will only contribute to the mass of the gravitational fluctuation.
The background evolution is governed by the inflaton field ϕ with the following metric

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2). \tag{8}$$

After the end of inflation, the inflaton start to oscillate which is known to be responsible for
the reheating of our universe. For current study we will consider the solution of the oscillating
inflaton from our reference [17] as shown in the fig.1. As we have emphasized before, our
mechanism does not depend on the particular model of inflation under consideration. As
long as we have oscillating background inflaton, we will have broad parametric resonance
for the gravitational fluctuation for a wide range of momentum modes depending upon the
oscillation frequency of the inflaton and the coupling paramter λ. At the linear order, the
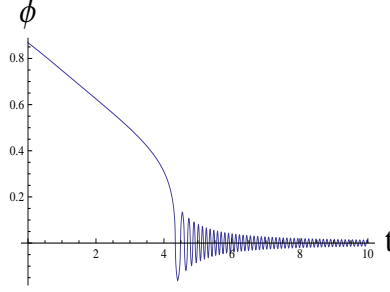scalar field fluctuation will follow the same dynamics as before. Since our mechanism of

FIG. 1: Background evolution of the inflaton field ϕ in cosmological time t. We measure the value of ϕ in unit of f and cosmic time t in unit of s.

pre-heating is mainly dependent on the dynamics of gravitational fluctuation, we consider tensor perturbations as follows

$$ds^2 = -dt^2 + a(t)^2(\delta_{ij} + h_{ij})dx^i dx^j \tag{9}$$

Apart from inflaton we only have two tensor polarization degrees of freedom. Therefore, we can always choose the following transverse and traceless gauge condition for the tensor perturbation,

$$\partial_i h^{ij} = 0 \; ; \; \delta_{ij} h^{ij} = 0. \tag{10}$$

The fourier mode function of h_{ij} defined as

$$h_{ij}(t,x) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \sum_{s=1,2} [e^s_{ij}(\mathbf{k})\tilde{h}^s(\mathbf{k},t)e^{i\mathbf{k}\cdot\mathbf{x}} + h.c.], \tag{11}$$

Where, $e^s_{ij}(\mathbf{k})$ are the two independent polarization tensor of the gravitational wave. To the linear order in cosmological perturbation theory, the Fourier transformed gravitational wave equation turns out to be

$$\ddot{\tilde{h}}^s + 3H\dot{\tilde{h}}^s + \left(\frac{k^2}{a^2} + m(\phi)^2\right)\tilde{h}^s = 0. \tag{12}$$

This equation is modified Mathieu equation with the oscillating inflaton background. The usual re-heating phenomena is based on this same form of equation of a matter field. Analytic study of this equation has been done quite extensively in the context of pre-heating. The important difference here is that, resonant production of gravitational wave initiate the pre-heating process through the minimal coupling of the matter field.

By using the properties of the gravitational wave, the form of the scalar field equation becomes

$$\ddot{\theta} + 3H\dot{\theta} - \frac{1}{a^2}\nabla^2\theta + \frac{h^{ij}}{a^2}\partial_i\partial_j\theta + m_\theta^2\theta = 0. \tag{13}$$

7

As one notices that in order to have gravitational effect we restrict up to the second order term. Considering the following Fourier transformation for the scalar field,

$$\theta(t, x) = \int \frac{d^3\mathbf{k}}{(2\pi)^{2/3}} [a_\mathbf{k} \, \tilde{\theta}(\mathbf{k}, t)e^{i\mathbf{k}\cdot\mathbf{x}} + h.c.], \tag{14}$$

the mode equation for $\tilde{\theta}(\mathbf{k})$, take the following from

$$\ddot{\tilde{\theta}} + 3H\dot{\tilde{\theta}} + \left(\frac{k^2}{a^2} + m_\theta^2\right)\tilde{\theta} = -\left(\frac{2}{\pi}\right)^{3/2} \frac{1}{a^2} \int \sum_s k_i' k_j' \, e_{ij}^s(\mathbf{k} - \mathbf{k}')\tilde{h}^s(\mathbf{k} - \mathbf{k}') \, \tilde{\theta}(\mathbf{k}') \, d^3\mathbf{k}'. \tag{15}$$

In the integral part of the above evolution equation for the scalar mode, we consider the evolution of a particular scalar mode of $\mathbf{k}$. For a fixed vector $\mathbf{k}$, one can parametrize the two tensor polarization $e_{ij}^s(\mathbf{q})$ modes of the gravitational wave in the above integral as,

$$\begin{aligned}
e_1^{ij}(\mathbf{q}) &= \epsilon_1^i(\mathbf{q})\epsilon_2^j(\mathbf{q}) + \epsilon_2^i(\mathbf{q})\epsilon_1^j(\mathbf{q}), \\
e_2^{ij}(\mathbf{q}) &= \epsilon_1^i(\mathbf{q})\epsilon_1^j(\mathbf{q}) - \epsilon_2^i(\mathbf{q})\epsilon_2^j(\mathbf{q}).
\end{aligned}$$

where, in a particular basis we write

$$\begin{aligned}
\boldsymbol{\epsilon}_1(\mathbf{q}) &= \left\{-\frac{\mathbf{q}_y}{q_{xy}}, \frac{\mathbf{q}_x}{q_{xy}}, 0\right\}, \\
\boldsymbol{\epsilon}_2(\mathbf{q}) &= \left\{\frac{\mathbf{q}_x\mathbf{q}_z}{q_{xy}q}, \frac{\mathbf{q}_y\mathbf{q}_z}{q_{xy}q}, -\frac{\mathbf{q}_{xy}}{q}\right\},
\end{aligned} \tag{16}$$

considering $\mathbf{q} = (\mathbf{k}' - \mathbf{k})$. As we have mentioned before, for a fixed scalar mode $\theta_\mathbf{k}$ with momentum $\mathbf{k} = k\hat{\mathbf{z}}$, if we parametrize the $\mathbf{q} = \{k'\sin\alpha\cos\beta, k'\sin\alpha\sin\beta, k - k'\cos\alpha\}$. With the above parametrization, the equation of motion for the scalar mode function turned out to be

$$\ddot{\tilde{\theta}} + 3H\dot{\tilde{\theta}} + \left(\frac{k^2}{a^2} + m_\theta^2\right)\tilde{\theta} = 2\pi\left(\frac{2}{\pi}\right)^{3/2}\frac{k^2}{a^2}\int \frac{k'^4\sin^3\alpha \, \tilde{h}(\mathbf{k} - \mathbf{k}')}{k^2 + k'^2 - 2kk'\cos\alpha}\tilde{\theta}(\mathbf{k}') \, d\alpha dk'. \tag{17}$$

Now this is a modified Mathieu equation, where the oscillatory background is set by all possible modes of the gravitational wave $\tilde{h}(\mathbf{k} - \mathbf{k}')$, which satisfy the following equation ,

$$\ddot{\tilde{h}}^s + 3H\dot{\tilde{h}}^s + \left(\frac{k^2 + k'^2 - 2kk'\cos\alpha}{a^2} + m(\phi)^2\right)\tilde{h}^s = 0. \tag{18}$$

In order to understand the behaviour of these equations, we will consider two different limit on the possible value of the scalar mode momentum. First let us consider large k limit compared to k'. The modes in this range will contribute the most. In this limit, the solution of $\tilde{h}(\mathbf{k} - \mathbf{k}')$ becomes approximately independent of k'. Therefore, the above equation for

the scalar mode function will take the following simplified form after performing the angular integral,

$$\ddot{\tilde{\theta}} + 3H\dot{\tilde{\theta}} + \left(\frac{k^2}{a^2} + m_\theta^2\right)\tilde{\theta} = \frac{8\pi}{3}\left(\frac{2}{\pi}\right)^{3/2}\frac{\tilde{h}(\mathbf{k})}{a^2}\int k'^4\,\tilde{\theta}(\mathbf{k}')\,dk'. \qquad (19)$$

To understand the scale around which we will have the resonant amplification within the perturbative limit, let us introduce the natural time scale s^{-1} coming from the period of oscillation of the inflaton field after the end of inflation as mentioned in fig.1. If we measure our time in term of s unit, then the above scalar mode equation turns out to be

$$\ddot{\tilde{\theta}} + 3H\dot{\tilde{\theta}} + \left(\frac{k^2}{s^2 a^2} + \frac{m_\theta^2}{s^2}\right)\tilde{\theta} = \frac{8\pi}{3}\left(\frac{2}{\pi}\right)^{3/2}\frac{s^3\tilde{h}(\mathbf{k})}{a^2}\int \frac{k'^4}{s^4}\,\tilde{\theta}(\mathbf{k}')\,\frac{dk'}{s}. \qquad (20)$$

For the purpose of our analytic discussion, we set the scale factor $a(t) = 1$. This is a reasonable assumption during pre-heating, as the scale factor remains almost constant. We also use this fact for our numerical analysis in the later section. At this point we also would like to point out that the combination $s^3\tilde{h}(\mathbf{k})$ is dimensionless as can bee seen from the fourier expansion of the gravitational wave eq.(11). Therefore, our perturbative analysis will be under control as long as one satisfies $s^3\tilde{h}(\mathbf{k}) < 1$. From the usual properties of Mathieu equation, the broad parametric resonance will occur if the coefficient of $\tilde{\theta}(\mathbf{k}')$ in the integral part of the above scalar mode equation is larger than unity. Therefore, from the scaling properties of momentum k' and $\tilde{h}(\mathbf{k})$, we can set the following qualitative condition between the magnitude of the gravitational field and the momentum modes contributing to the scalar particle production as follows,

$$s^3\tilde{h}(\mathbf{k})\left(\frac{k'}{s}\right)^5 \gg 1 \implies \left(\frac{s}{k'}\right)^5 \ll (s^3\tilde{h}(\mathbf{k})) \qquad (21)$$

Furthermore, within the perturbative regime, the modes which will contribute to the broad parametric resonance for the scalar amplitude should satisfy the following condition,

$$\left(\frac{s}{k'}\right)^5 \ll s^3\tilde{h}(\mathbf{k}) < 1. \qquad (22)$$

However, if we consider, k' to be larger than k, following the same procedure we have discussed so far, the equation of motion for the scalar mode function will turn out to be

$$\ddot{\tilde{\theta}} + 3H\dot{\tilde{\theta}} + \left(\frac{k^2}{s^2 a^2} + \frac{m_\theta^2}{s^2}\right)\tilde{\theta} = \frac{8\pi}{3}\left(\frac{2}{\pi}\right)^{3/2}\frac{k^2}{s^2 a^2}\int \frac{k'^2}{s^2}\,s^3\tilde{h}(\mathbf{k}')\tilde{\theta}(\mathbf{k}')\,\frac{dk'}{s}. \qquad (23)$$

Following the similar argument as we have discussed so far, the constraint on the momentum modes will be as follows

$$\left(\frac{s}{k}\right)^2\left(\frac{s}{k'}\right)^3 \ll s^3\tilde{h}(\mathbf{k}') < 1, \quad \text{for} \quad k \ll k'. \qquad (24)$$

At this point would like to emphasized the fact that the conditions for the momentum modes eqs.(22) and eq.(24) for two different limit of the scalar momentum $k >> k'$ and $k << k'$ respectively, are within the perturbative limit. The usual parametric resonance will lead the gravitational amplitude to be in the non-perturbative regime. In our present analysis we will confine ourselves in the perturbative regime by limiting the amplitude of the gravitational wave to be less than unity. However, non-linear analysis must be needed to understand complete dynamics of the system. Furthermore, it may also be possible that back-reaction of the scalar particle production into the gravitational wave dynamics keep it always in the perturbative regime. We defer all these issues for our future studies.

We have not introduced any direct coupling of the matter field with the inflaton. Therefore, it important to note that in the dynamics of the matter field θ, the gravitational fluctuation will play the role at the first non-linear order. However, in order to realise our mechanism, not only we need to choose suitable mass term $m(\phi)$ for the graviton fluctuation, but also we need to be careful about its effect during the inflation. In the usual massive gravity theory, modification is tractable in the infra-red limit as the theory remains perturbative in nature. Non-linearity of the theory depends crucially upon the mass term of the graviton. In the current construction, we consider gravitational mass is dependent on the inflaton field itself. Therefore, one needs to study the strong coupling aspects of the theory which we will defer for our future study. Motivated by the reference [9] we assume, in order not to have large power suppression of the gravitational wave during inflation,

$$m(\phi)^2 = \frac{\lambda \phi^2}{1 + \left(\frac{\phi}{\phi*}\right)^4}, \tag{25}$$

where $\phi*$ is the value of ϕ at the end of inflation. Such a functional form will lead to very small mass of the gravitational wave during inflation, since inflaton field will have very large constant vacuum expectation value during inflation. One can easily compute the tensor power spectrum during inflation with almost constant mass limit of the graviton to be

$$P_h = \frac{2H^2}{\pi^2 M_p^2} \left(\frac{k}{aH}\right)^{\frac{2m(\phi)^2}{3H^2}}. \tag{26}$$

Therefore, as one can see that due to the specific choice of our mass funtion $m(\phi)$, modified part of the tensor power spectrum due to the mass term is highly suppressed. However, after the end of inflation, since the inflaton undergoes damped oscillation around the minimum of it's effective potential, the amplitude of the inflaton will be very small compared to $\phi*$.
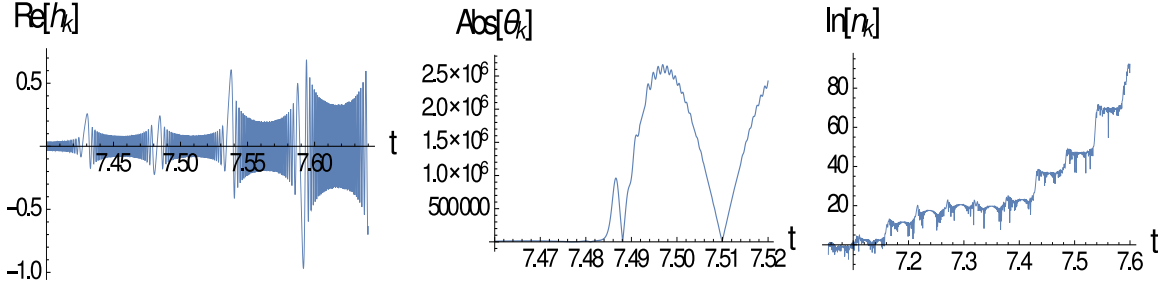
FIG. 2: Behaviour of gravitational wave $Re[h_k]$ in unit of s^3, the matter field θ_k and particle number density n_k in time t which is again measured in unit of s. We have plotted for $k_x = 0, k_y = 170, k_z = 130$ in unit of s. We chosen the dimensionless graviton mass parameter $\lambda = 1$. We also checked that as we decrease the initial amplitude of the gravitational wave, the resonant production of the gravitational wave happens at later time which leads to the particle production at late time.

Therefore, during oscillating phase, we can approximate,

$$m(\phi)^2 = \lambda\phi^2, \tag{27}$$

where λ is dimensionless coupling constant. observing the above expression for the inflaton dependent mass term, one would expect to get back the usual massless gravity theory long after the end of inflation. For a class of massive gravity theory [7]under consideration, it has been shown that it reproduces usual gravity theory in the massless graviton limit. In this paper we will be restricted to a particular class of minimal massive gravity models where the number of degrees of freedom is three.

Now considering the gravitational wave equation with the above form of the gravitational mass term

$$\ddot{\tilde{h}}^s + 3H\dot{\tilde{h}}^s + \left(\frac{k^2}{s^2 a^2} + \frac{\lambda\phi^2}{s^2}\right)\tilde{h}^s = 0. \tag{28}$$

As usual, we consider the time scale in unit of s. As has been extensively discussed in [6], and we also have observed from our numerical analysis the successive enhancement of the amplitude of the gravitational wave happens at the time of zero crossing of the inflaton field, and it oscillates with almost the same amplitude in between the two successive crossings. Therefore, analytic treatment of this equation suggested in [6], can be done by

11

the method of successive parabolic scattering. The stochastic resonance certainly will lead the perturbative gravitational amplitude into the non-perturbative regime. Therefore, one needs to go the non-linear regime. However, in the present paper we will restrict our study in the perturbative regime such that the approximate energy density of the gravitational wave [6]

$$\rho_h \simeq \frac{\lambda|\phi|}{\pi^2 a^3} \int_0^\infty dk \; k^2 \; |\beta_k|^2, \tag{29}$$

where, β_k is the amplitude of the negative frequency part of the full solution, is smaller than the inflaton energy density. Because of resonance, the approximately expression for $|\beta_k|^2 \simeq \frac{1}{2}e^{2\mu_k st}$, where the exponent μ_k, defined for a particular instant of zero crossing of the inflaton field is given as

$$\mu_k(t) = \frac{1}{2\pi}ln\left(1 + 2e^{-\kappa^2} - 2\sin\theta e^{-\frac{\kappa^2}{2}}\sqrt{1 + e^{-\kappa^2}}\right). \tag{30}$$

The expression for $\kappa^2 = \frac{\pi k^2}{a^2 \lambda s \phi(t)}$, and θ is random phase within $(0, 2\pi)$. Therefore, on can see the exponential enhancement of the gravtitional field amplitude. In order to be in the perturbative regime, we have to consider ts begin very small.

One may notice, in our current analysis the eq.(36),in the dynamics of the gravity wave, we ignored the back-reaction of the matter field. It can be easily understood that at least at the initial stage of the pre-heating process, the back-reaction should be small. But it must be very important after sufficient number of matter particle produced. The full analysis taking into account the back-reaction and rescattering of the particles will be considered in our subsequent publication. However, as we pointed out, the matter particle production will happen at the non-linear level. Thanks to the parametric resonance that has happened in the gravitational sector. Because of this resonance, we will show, how efficient the particle production is in our mechanism compared with the usual reheating mechanism. In order to quantify the amount particles that has been produced for θ_k with momentum $\mathbf{k}$ we use the following well known expression of particle number density [6]

$$n_k = \frac{\omega_k}{2}\left(\frac{|\dot{\tilde{\theta}}(\mathbf{k})|^2}{\omega_k^2} + |\tilde{\theta}(\mathbf{k})|^2\right) - \frac{1}{2}. \tag{31}$$

Where, ω_k is the time dependent frequency of a particular mode $\mathbf{k}$.

$$\omega_k = \sqrt{\frac{k^2}{a^2} + m_\theta^2} = \sqrt{k_{phy}^2 + m_\theta^2}, \tag{32}$$

where k_{phy} is the physical momentum. In the above derivation of the number density, we have used the following canonical quantization condition for the scalar field mode,

$$\dot{\tilde{\theta}}^*(t)\tilde{\theta}(t) - \tilde{\theta}^*(t)\dot{\tilde{\theta}}(t) = i \qquad (33)$$

In the next section will study numerically two equations of motion and try to compute the particle number density for a particular mode. For simplicity of our numerical computation, we fix the value of k_x, k_y and do the numerical integration for the mode equation of θ_k along the k_z.

A. Numerical analysis and particle production

As we have already discussed before, we are only interested in the oscillatory regime of the inflaton, our final conclusion does not really depend on a specific model of inflation. In order to get the background of oscillatory inflaton for our numerical calculation, we consider our specific model of modified natural inflation [17]. We choose $M(\phi) = \sin^7 x \left[1 - \cos x \sin^2 x\right]^{14}$, where $x = \phi/f$. For this specific form of the function, we had the number of e-folding $\mathcal{N} = 50$, and the value of the spectral index is $n_s = 0.96$. In order to achieve the aforementioned cosmological quantities compatible with the observations, we found the values of the other parameters to be $f = 0.84 M_p, s = 2.25 \times 10^{-6} M_p, \Lambda = 0.011 M_p$. For our numerical purpose we set $t = 0$ as the beginning of the inflation. After the end of inflation at around $t \sim 4$ cosmic time in unit of s, the inflaton field will start to have coherent damped oscillation as shown in the fig-1. At this point, we are again emphasizing the fact that our analysis and the final result is confined within the oscillatory regime of the inflaton field. The oscillation after inflation is generic to any inflationary model. Therefore, our qualitative results are insensitive to the specific model of inflation under consideration. For the sake of our numerical analysis, at time $t = 6$ we normalize the cosmological scale factor $a(6) = 1$. For a wide range of initial amplitude of the gravitational wave, we found the resonant production of the gravitational wave. More over this resonant production of gravitational wave will trigger the pre-heating phase of the universe through the matter fields with the universal gravitational coupling. In the inflationary background, to quantize a field we choose the usual Bunch-Davis vacuum,

$$\tilde{\theta}(k) = \lim_{t \to 0} \frac{1}{\sqrt{2k}} e^{-ik \int \frac{1}{a(t)} dt} \left(1 + \frac{i}{k \int \frac{1}{a(t)} dt}\right). \qquad (34)$$
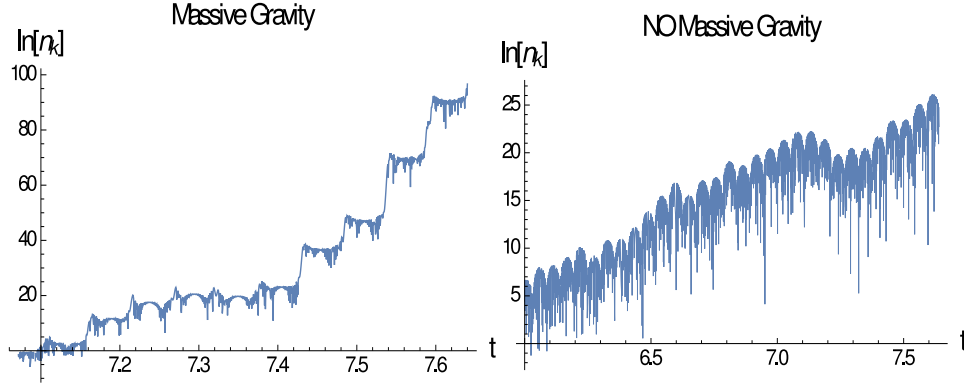
FIG. 3: Comparing the particle production for matter field $\tilde{\theta}(k)$ in massive gravity scenario (left panel) with the usual pre-heating scenario(right panel) considering $\mathcal{L}_{int} \sim g \, \phi^2 \, \theta^2$ coupling term. Plot for the particle density n_k in time t which is measured in unit of s. We take $k_x = 0, k_y = 170, k_z = 130$ in unit of s. Choose $\lambda = g = 1, m_\theta = 0.1s$.

In the above expression, we also assume approximately the de-Sitter vacuum. As we see, lower the value of a particular mode k, larger the initial amplitude in the quantum vacuum, leads to the earlier production of matter particle of that particular modes after the end of inflation. As a particular mode starts evolving from its quantum vacuum defined in the far past and deep inside the Hubble volume during inflation, its amplitude will start to decrease due to Hubble friction and finally gets frozen after the horizon exit. Therefore, during reheating the amplitude at the horizon exit will set the initial condition for a particular mode of the gravitational wave to evolve. From the aforementioned arguments, we see that the parametric resonance for the matter field will happen mostly for the long wavelength modes compared to the Hubble length. We can clearly see from the fig-2 that because of the resonant gravitational wave production triggered by its oscillatory mass term, minimal coupling of the matter field with the gravity is sufficient to reheat the universe.

we also want to compare our result with respect to the usual pre-heating scenario [6]. At the linear order in perturbation theory, in our current scenario we want to see how efficient the particle production is in comparison with the usual pre-heating scenario [6]. One can clearly see the difference between the distinct effect of parametric resonance on the gravitational fluctuation $\tilde{h}(k)$ and the matter field fluctuation $\tilde{\theta}(k)$ from the fig.2. Important point to mention that in the scalar field sector, the parametric resonance happens in the resonant

14

gravitational field background. Therefore, one can clearly see from the left panel of Fig.3 that the produced density of particle in the massive gravity framework is behaving like double exponential function in time. The initial condition, we have used in our analysis is that there in no particle at $t = 6$ cosmic time in unit of s after the inflation. After the end of cosmic time $t = 7.64$, so that the amplitude of the gravitational wave is in the perturbative limit, one can see from the fig.3 that the number of particle produced per unit volume in the massive gravity scenario is $\sim e^{75}$ times larger than that of the usual pre-heating scenario for a particular mode. Therefore, in the massive gravity scenario, the particle production is much more efficient compared to the usual pre-heating scenario. It is possible that most of the energy of the inflaton field may easily be transferred through successive parametric resonance first to the gravitational sector and then to the matter sector within a very short period of time, and the usual reheating follows. The whole mechanism of particle production that we have discussed so far is highly non-equilibrium in nature. Therefore, the elementary theory of reheating is still necessary to thermalization all the produced particle, and the universe to enter into the usual radiation phase. Now, because of highly efficient particle production compared to the usual pre-heating scenario, if most of the inflaton energy is transferred to the relativistic matter particle, the usual standard model couplings among the produced particles may be sufficient to thermilize them. Therefore, we may not need to introduce any arbitrary coupling at all to have the successful reheating of our universe after the inflation. Of course our current analysis is in the linear regime, and we have not taken into the back-reaction and the re-scattering of the fields. We differ the detail studies of all the aforementioned points for our future work.

IV. CONCLUSION

In this paper, we have tried to constructed a phenomenological model of pre-heating by introducing inflaton field dependent mass term for a certain class of massive gravity theory. In that particular class of theories, we have three degrees of freedom because of some special internal symmetries. For the purpose of our study, we identified the extra degree of freedom to be the inflaton field. During inflation, gravitational mass term will suppress the power in the gravitational wave spectrum. In order to avoid large suppression, we parametrize the mass function in such a way that during inflation, its value is very small. On the other hand, after the end of inflation, the amplitude of the gravitational wave will be amplified

due to parametric resonance because of the mass term[9]. Now the question naturally arises is why have not we observed this large amplitude gravitational wave yet? Natural answer would be that this large amplitude gravitational wave will contribute largely to reheat our universe after the inflation. In this paper we have shown that how this resonant gravitational wave background can naturally lead to the pre-heating phase of our universe. Interestingly, to pre-heat the universe we do not need to introduce any arbitrary couplings between the visible sector matter field and the inflaton field in an *ad hoc* manner. The natural universal gravitation coupling is sufficient to reheat the universe. At the linear order in perturbation theory, we also compared the efficiency of the particle production between our mechanism and the usual reheating mechanism [6]. We found that because of the resonant gravitational background, in our mechanism the particle production is highly efficient compared to the usual scenario. This phenomena of resonance in a resonant background has not been studied before at least in the cosmological context. It would very much interesting to study this mechanism in detail in various physical situations.

Using the same mechanism that we discussed so far, we can study the production of chiral gravitational wave during preheating if we further introduce the Chern-Simons (CS) higher derivative term [18, 19] coupled with the inflaton field in the action, i.e.

$$\mathcal{L}_{CS} \simeq \frac{g}{f} \phi R \tilde{R}. \tag{35}$$

The inflaton field is now like a pseudo scalar axion, where

$$R\tilde{R} = \tilde{R}^{\nu}{}_{\nu}{}^{\alpha\beta} R^{\nu}{}_{\mu\alpha\beta} \quad ; \quad \tilde{R}^{\nu}{}_{\nu}{}^{\alpha\beta} := \frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} R^{\mu}{}_{\nu\gamma\delta}.$$

$\epsilon^{\mu\nu\alpha\beta}$ is the usual 4-dimensional Levi-Civita tensor. g is the free coupling parameter. Since in the background of inflaton this term violates parity, the left and right handed gravitational wave propagate differently according to the following equation

$$\left(M_p^2 + \frac{k\,g\,\eta_A}{f\,a} \dot{\phi} \right) \left(\ddot{h}_A^k + 3H\dot{h}_A^k + \frac{k^2}{a^2} h_A^k \right) + m(\phi)^2 h_A^k$$
$$= -\frac{k\,g\,\eta_A}{f\,a} (\ddot{\phi} - H\dot{\phi})\, \dot{h}_A^k, \tag{36}$$

where $A = L, R$ are left and right circular polarization mode of the gravitational wave. Therefore, we will have resonant chiral gravity wave production. This effect may be very important to produce the leptogenesis in the early universe. As is well know [20], if we couple gravity with the leptons, the total lepton number current will no longer be conserved

because of gravitational anomaly,

$$\partial_\mu j^\mu = \frac{3}{16\pi^2} < R\tilde{R} >, \tag{37}$$

where, $< R\tilde{R} >$ is the quantum expectation value of the Chern-Simons term. Therefore, non-zero expectation value of this Chern-Simons operator will produce the leptogenesis. This mechanism has already been discussed in [18]. However, in our model the resonant production of chiral gravitational wave will happen during oscillation period of the inflaton. Therefore, the maximum lepton number violating contribution will be coming during the preheating period. Currently we are looking into this issue in greater detail. We are also looking into the possibility this new reheating mechanism in a Lorentz invariant higher derivative gravity theory coupled with inflaton field. We know that the higher derivative term such as Gauss-Bonnet term coupled with the inflaton field contributes an effective inflaton field dependent mass term in the gravitational fluctuation. Currently we are also looking into that possibility.

We have also pointed out that the specific class of massive gravity theories, we have considered for our study, need to be studied in detail. In massive gravity theory, one generally introduces the constant mass term for the graviton. But here we have introduced extra field dependent mass term [9]. This particular field dependent mass term should be studied with a special care. As one knows interacting massive spin two field coupled with other field such as our scalar field has several constraints to be satisfied. We keep all these questions for our future studies.

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[1] P. A. R. Ade et al. [Planck Collaboration], arXiv:1303.5082 [astro-ph.CO]

[2] A. H. Guth, Phys.Rev. D **23**, 347 (1981).

[3] A. D. Linde, Phys. Lett. B **108**, 389 (1982).

[4] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 12201223 (1982).

[5] J. H. Traschen and R. H. Brandenberger, Phys. Rev. D **42**, 2491 (1990); Y. Shtanov, J. Traschen, and R. Brandenberger, Phys. Rev. D **51**, 5438 (1995), arXiv:hep-ph/9407247 .

[6] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. Lett. **73**, 3195 (1994), arXiv:hep-th/9405187; L. Kofman, A. Linde, and A. A. Starobinsky, Phys. Rev. D **56**, 3258

(1997), arXiv:hep-ph/9704452 .

[7] S. L. Dubovsky, JHEP 0410, 076 (2004); S. L. Dubovsky, P. G. Tinyakov and I. I. Tkachev, Phys. Rev. Lett. 94, 181102 (2005) [hep-th/0411158].

[8] S. Alexander, *et al.*, arXiv:1405.4288

[9] C. Lin and Misao Sasaki, arXiv:1504.01373 [astro-ph.CO]

[10] D. H. Lyth, Phys. Rev. Lett. 78, 1861 (1997) [hep-ph/9606387]; R. Easther, W. H. Kinney and B. A. Powell, JCAP **0608**, 004 (2006); S. Antusch, D. Nolde, JCAP **1405**, 035 (2014) arXiv:1404.1821.

[11] M. Fierz and W. Pauli, Proc. Roy. Soc. Lond. A173 (1939) 211232.

[12] C. de Rham, G. Gabadadze and A. J. Tolley, Phys. Rev. Lett. 106, 231101 (2011) [arXiv:1011.1232 [hep-th]].

[13] G. D'Amico, C. de Rham, S. Dubovsky, G. Gabadadze, D. Pirtskhalava and A. J. Tolley, Phys Rev. D **84**, 12404 (2011); A.E. Gumrukcuoglu, C. Lin and S. Mukohyama, JCAP **1111**, 030 (2011); N. Khosravi, G. Niz, K. Koyama and G. Tasinato, JCAP **1308**, 044 (2013); A. De Felice, A.E. Gumrukcuoglu, C. Lin and S. Mukohyama, JCAP **1305**, 035 (2013); S. Deser, A. Waldron, Phys. Rev. Lett. **110**, 111101 (2013) ; S. Deser, K. Izumi, Y. C. Ong and A. Waldron, Phys. Lett. B **726**, 544 (2013).

[14] C. de Rham, Living Rev. Rel. **17**, 7 (2014) [arXiv:1401.4173];

[15] V. A. Rubakov, hep-th/0407104; D. Comelli, F. Nesti and L. Pilo, JHEP **1307**, 161 (2013) [arXiv:1305.0236 [hep-th]]; C. Lin, Phys. Lett. B **727**, 31 (2013) [arXiv:1305.2069]; C. Lin, Phys. Lett. B **738**, 386 (2014) [arXiv:1307.2574]; C. Lin and L. Z. Labun, arXiv:1501.07160 [hep-th].

[16] D. Comelli, F. Nesti and L. Pilo, JCAP **11**, 018 (2014) [arXiv:1407.4991 [hep-th]];

[17] D. Maity, and P. Saha, Phys.Rev. D **91**, 023504 (2015); D. Maity, Phys. Lett. B **720**, 389 (2013).

[18] S. H. S. Alexander, M. E. Peskin, and M. M. Sheikh- Jabbari, Phys. Rev. Lett. 96, 081301 (2006), arXiv:hepth/0403069.

[19] S. H. Alexander and J. Gates, S. James, JCAP 0606, 018 (2006), arXiv:hep-th/0409014 [hep-th]; V. Taveras and N. Yunes, Phys.Rev. D78, 064070 (2008), arXiv:0807.2652 [gr-qc]; S. Mercuri and V. Taveras, Phys.Rev. D80, 104007 (2009), arXiv:0903.4407 [gr-qc]; S. Weinberg, Phys.Rev. D77, 123541 (2008).

[20] L. Alvarez-Gaume and E. Witten, Nucl. Phys. B234, 269 (1984).